The effects of international shocks on Australia's business cycle*

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February 7, 2010

Abstract

This paper examines the sources of Australia's business cycle fluctuations. The cyclical component of GDP is extracted using the Beveridge-Nelson decomposition and a structural VAR model is identified using robust sign restrictions derived from a structural small open economy model. In contrast to previous VAR studies, international factors are found to contribute to over half of the output forecast errors whereas demand shocks have relatively modest effects.

Keywords: Australia business cycle, sign restriction VAR, stabilization policy and international shocks

JEL Classification: E32, E52 , E63, F41

*The author thanks Farshid Vahid, Jarkko Jaaskela, Kristoffer Nimark, Renee Fry, Chris Kent, David Vines, Heather Anderson, Warwick McKibbin and two anonymous referees for helpful comments and insightful discussions, and seminar participants at the University of Otago and ANU. The author's research has benefitted from support provided by ARC grant DP0664024.

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I Introduction

There is little consensus on the role played by the rest of the world in a small open economy's business cycle. In the case of Australia, Dungey (2002) estimates a structural vector autoregression (SVAR) model, which implies that international factors account for 32 per cent of output forecast errors over a one year horizon, while domestic GDP shocks remain the dominant contributor. A SVAR model for Australia by Brischetto and Voss (1999) reveals that only around 5 per cent of output forecast errors stems from foreign factors. On the other hand, using an estimated New Keynesian dynamic stochastic general equilibrium (DSGE) model, Nimark (2007) concludes foreign shocks explain over 50 per cent of the variance in Australian output around its trend while domestic output shocks account for only 8 per cent. Using a different criteria, Dungey and Pagan (2000) simulate data from a SVAR model and find that recessions would have been less severe in the absence of foreign disturbances, while cumulated movements during the expansion phrase would also have been smaller.

This paper argues that the different findings of the studies cited above can be understood as resulting from the difficulty of deciding how to appropriately identify the structural disturbances relevant to a small open economy. Identifying restrictions for SVAR models may introduce substantial misspecifications that could lead to invalid inference. At the same time, identification of structural disturbances by means of cross-equation restrictions from a small DSGE model may be a too stringent method to capture the complex dynamics of the data generating process. This paper contributes to this debate by developing a VAR model of the Australian economy using robust sign restrictions derived from an estimated DSGE model. One key element of this approach is that it allows for a theoretically consistent view of the relationships between the set of macro variables without imposing the full DSGE structure.

Earlier sign restriction VAR studies focus mainly on identifying a subset of structural disturbances, examples include Faust (1998) and Uhlig (2005) who identify only monetary policy shocks. More recent studies by Canova and De Nicolo (2002) and Peersman (2005) apply the sign restriction methodology to identify all shocks in the VAR model. All these studies, however, are based on large economies with little discussion of the role of exchange rates. One exception is Farrant and Peersman (2006), who investigate the role of exchange rates in an open economy setting. However, the role of international factors is not explicitly discussed in that study.

The use of restrictions derived from a theoretical model to aid VAR estimation is not new. McKibbin, Pagan, and Robertson (1998) use the McKibbin-Sachs Global (MSG2) model to restrict the long-run behaviour of a VAR, while the short-run features are left unrestricted. Dungey and Pagan (2009) try to reconcile their earlier SVAR model with restrictions implied by a simple open economy DSGE model. Peersman and Straub (2004) use a calibrated real business cycle (RBC) model to derive sign restrictions in order to identify technology shocks.

The starting point of this paper is to use the Beveridge-Nelson decomposition to extract the cyclical component of GDP, which will be used as a measure of Australia's business cycle. A slightly modified version of the small open economy model proposed in Monacelli (2005), and Gali and Monacelli (2005) is then estimated using maximum likelihood. The estimated model is used to determine a set of robust sign restrictions for the VAR analysis. The small open economy assumption is imposed on the VAR model by restricting the impact of domestic variables on foreign variables. The ultimate aim of the analysis is to map the set of statistical relationships estimated from the reduced form VAR back into a set of structural disturbances for economic interpretation. To do this, an algorithm similar to that proposed by Canova and De Nicolo (2002) is used to trace out all possible orthogonal vector moving average (VMA) representations of the VAR that are consistent with the sign restrictions derived from the estimated DSGE model. Since there is not enough information to uniquely identify a set of structural disturbances, the median impulse approach suggested in Fry and Pagan (2005) is used to summarise the results.

The analysis reveals several interesting results. First, the Beveridge-Nelson decomposition produces a plausible measure of Australia's output fluctuations. The characteristics of the cyclical behaviour match previous business cycle studies using factor models such as Gillitzer, Kearns, and Richards (2005). Second, in contrast to previous SVAR studies for Australia, foreign factors account for over half of the output forecast errors whereas innovations from output itself have only a modest effect. The result is robust across different foreign specifications using data for the United States and the Group of Seven (G7) Countries.

The rest of the paper is organised as follows. Section 2 describes the Beveridge-Nelson decomposition used to extract the cyclical component of GDP. Section 3 outlines the estimated small open economy DSGE model together with the data used in the analysis. A set of robust sign restrictions is derived from the estimated DSGE model for the open economy VAR. Section *IV* describes the estimation and identification of the open economy sign restriction VAR model. Section *V* summarises the estimation results. Finally, Section *VI* reviews the main findings.

II The Cyclical Component of GDP

The first step of the analysis of this paper is to obtain a measure of the cyclical component of GDP. The cyclical component is defined as the difference between actual and the permanent component of GDP.[1] The permanent component is exctracted by means of a Beveridge-Nelson (BN) decomposition, which is preferred to one popular alternative, the Hodrick-Prescott (HP)-filter as the BN-decomposition allows for correlation between the innovations to the permanent and cyclical components.

A time series y_t with an ARIMA(p,1,q) representation can be decomposed into a permanent (τ_t) and cyclical (c_t) component using the BN decomposition as follows:

$$y_t = \tau_t + c_t, \tag{1}$$

where $\tau_t = \mu + \tau_{t-1} + \alpha\epsilon_t$ is the unobserved permanent component, which is assumed to follow a random walk with an average growth rate of μ; and $c_t = \phi_p(L)c_t + \psi_q(L)\epsilon_t + (1-\alpha)\epsilon_t$ is a stationary and invertible ARMA(p,q) process, where $\phi_p(0) = 0$ and $\Psi_q(L) = 0$.

TABLE 1
Test results: AIC, BIC and R^2

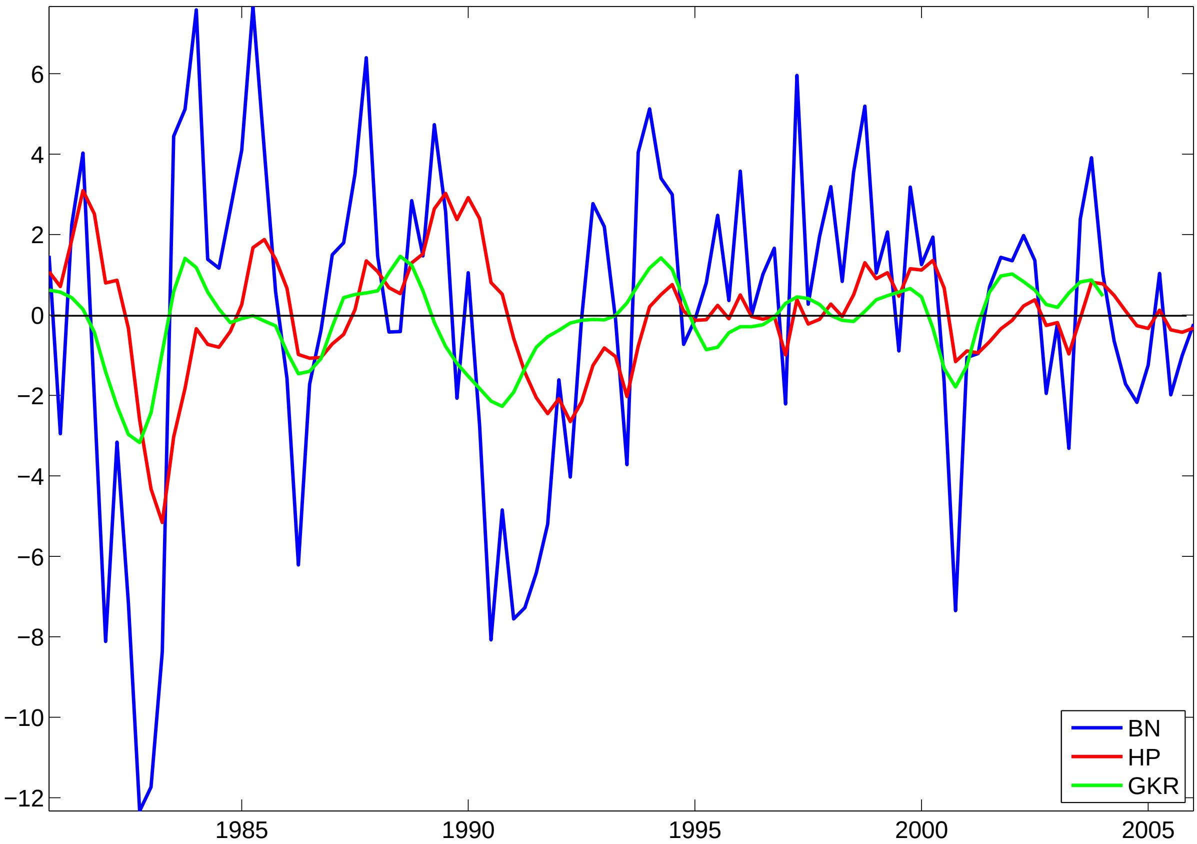
Model	Parameter estimates	AIC	BIC	R^2
ARIMA(1,1,1)	$c_t = 0.14c_{t-1} + 0.17\epsilon_{t-1} + \epsilon_t$	-6.828	-6.752	0.098
ARIMA(2,1,0)	$c_t = 0.30c_{t-1} + 0.01c_{t-2} + \epsilon_t$	-6.870	-6.769	0.094
ARIMA(2,1,1)	$c_t = -0.62c_{t-1} + 0.24c_{t-2} + 0.64\epsilon_{t-1} + \epsilon_t$	-6.825	-6.728	0.151
ARIMA(2,1,2)	$c_t = -0.35c_{t-1} + 0.42c_{t-2} + 0.72\epsilon_{t-1} - 0.27\epsilon_{t-1} + \epsilon_t$	-6.854	-6.749	0.154

Both the Akaike information and Schwarz criterion suggest that an ARIMA(2,1,1) model provides the best empirical fit for Australian quarterly real GDP between 1980Q4 and 2006Q1. The BN decomposition is computed based on the method suggested by Newbold (1990). The results for four alternative models are summarised in Table 1. Figure 1 shows that the BN-cycle is more volatile than the cycle derived using the HP–filter (based on the smoothing parameter

[1] The terms permanent component and trend are used interchangeably, as are cyclical component and the cycle. A detailed review of various detrending methods can be found in Canova (1998).

$\lambda = 1600$). This is particularly so in the first half of the sample[2] which displays more pronounced cycles. The two cycles have a similar peak frequency (estimated using the periodogram) around 17 quarters over the sample, with the BN cycle containing noticeably more high frequency oscillations. Figure 1 also shows the coincident (GKR) index of Australian economic activity derived by Gillitzer, Kearns and Richards (2005) using a factor model. This index provides a plausible measure of the Australian business cycle using a large number of macroeconomic variables.

FIGURE 1
Cycles in Australian GDP



Note: GKR is the cycle derived by Gillitzer, Kearns and Richards (2005) using a factor model.

The three series each imply a different underlying model, the aim here is to compare and contrast the different cyclical behaviour across the three detrending assumptions rather than to judge which is the best method to use.[3] For all three series, the two recessions during the early 1980s and 1990s are apparent. The BN-cycle and GKR index coincide with respect to

[2]The standard deviation of the BN-cycle is 3.9 per cent compared with 1.4 per cent for the HP-cycle over the whole sample.

[3]Another approach is to explicitly estimate the permanent component using a structural vector error correction model (SVECM) as in Pagan and Pesaran (2008).

the timing of recessions, suggesting a bottoming out of economic activity around 1983Q1 and 1991Q1. The HP-cycle is a bit slower at picking up the recessions.[4] In addition, the BN-cycle identifies two episodes of weak economic activity over the sample period. The first, in 1986, coincides with Paul Keating's *Banana Republic* remark over concerns about Australia's foreign debt position, a sharp depreciation of the exchange rate and a downturn in household expenditure. The slowdown of the economy following the end of the Sydney Olympic games and the introduction of the goods and services tax in 2000 is also apparent.

III A Stylised Small Open Economy DSGE Model

This section presents the estimated small open economy DSGE model. The model is based on a slightly modified version of that proposed by Monacelli (2005) and Gali and Monacelli (2005). The key advantage of using this model is due to its simplicity and the model includes the set of variables that are crucial for small open economy studies. It also embeds the key theoretical linkages often found in larger small open economy DSGE models. Variants of the proposed model have been heavily used in other applied macroeconomic research. Recent examples include Lubik and Schorfheide (2005) looking at the role of the exchange rate in the central bank's reaction function and Dungey and Pagan (2009) compare the model structure to their earlier SVAR model (see Dungey and Pagan (2000)).

Del Negro and Schorfheide (2009) argue model misspecification, in a sense that the model imposes invalid restrictions on the moving average representation of the macroeconomic time series, remains the key challenge for using DSGE models in empirical policy studies. Although the specific quantitative predictions of DSGE models maybe questionable, the theoretical linkages embedded in DSGE model remains useful for the understanding of contemporaneous relationships among key macroeconomic variables. This is the key motivation for using some but not all of the DSGE model's predictions in the form of robust sign restrictions. The estimated model is simulated to provide these set of restrictions for the VAR analysis.

The model consists of an open economy IS equation and a Phillips curve incorporating imperfect exchange rate pass-through. The monetary authority sets interest rates according to

[4] The HP filter can be thought of as a two-step filter: in the first step it renders y_t stationary; in the second it smooths the resulting stationary series with asymmetric moving average (MA) weights, which can contribute to a delay in identifying the recessions.

a Taylor-type reaction function, while the exchange rate depends on the interest rate differential between the domestic and foreign economies. The variables for the rest of the world are taken to be exogenous processes. The open economy IS equation derived from the consumer's optimising problem is:

$$\begin{aligned} y_t = n_1 y_{t-1} + (1-n_1)E_t y_{t+1} - n_2(r_t - E_t\pi_{t+1}) \\ + n_3 E_t \Delta y^*_{t+1} - n_4 z_t + n_5 E_t \Delta\psi_{t+1}, \end{aligned} \tag{2}$$

where: $n_1, \dots, n_5$ are parameters;[5] y_t is the aggregate output gap; r_t is the nominal interest rate; π_t is the inflation rate; y^*_t is the foreign output gap; and z_t represents technology disturbances that follow an AR(1) process.[6] $\psi_t = (1-\gamma)s_t - q_t$ can be interpreted as the *law of one price gap* which measures the deviation of the domestic price of imported goods from the world price, where s_t is is the terms of trade, defined as export prices relative to import price, and q_t is the real exchange rate. A non-zero ψ_t implies imperfect exchange rate pass-through to import prices. The backward looking component, y_{t-1}, in the IS equation is motivated by the assumption of habit persistence in consumer preferences.

The open economy New Keynesian Phillips Curve (NKPC) derived by solving the firm's pricing decision can be written as:

$$\pi_t = g_1\pi_{t-1} + (1-g_1)E_t\pi_{t+1} + g_2 y_t + g_3\psi_t + \epsilon_{\pi,t}, \tag{3}$$

where $\epsilon_{\pi,t}$ represents a cost push shock. The Phillips curve is based on the assumption of monopolistically competitive firms, subject to pricing constraints (Calvo pricing and indexation). If $g_3 = 0$, Equation (3) collapses down to a familiar closed economy Phillips curve where inflation dynamics are partly driven by past and expected inflation, in addition to the output gap. The open economy dimension includes the effects from the exchange rate as an important part of the monetary policy transmission process.

[5]It is important to note that without further restrictions, it is not possible to separately identify the parameter η_4 and the variance of the unobserved stochastic shock z_t. At a conceptual level, this does not cause real difficulty in the estimation procedure employed here (based on Monte Carlo Markov Chain simulations rather than numerical optimisation), see An and Schorfheide (2007). However, it is important to keep in mind that these two parameters have limited structural meaning. Nevertheless, this will not affect the sign restrictions used for the VAR model.

[6]A positive innovation to technology will increase the potential output of the economy hence has a negative effect on the output gap.

The assumption of perfect capital markets yields the standard uncovered interest parity (UIP) condition (which links the expected exchange rate depreciation to the interest rate differential):

$$q_t = E_t q_{t+1} + (r_t - E_t \pi_{t+1}) - \left(r_t^* - E_t \pi_{t+1}^*\right) + U_{q,t}, \tag{4}$$

where $U_{q,t}$ is a time-varying risk premium that follows an AR(1) process.

The monetary authority is assumed to set the nominal interest rate according to a Taylor rule based on contemporaneous inflation and output as well as an interest rate smoothing term:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)[\phi_1 \pi_t + \phi_2 y_t] + \epsilon_{r,t}, \tag{5}$$

where $\epsilon_{r,t}$ represents a non-systematic deviation from the reaction function. To complete the description of the structural model, the terms of trade s_t, the foreign output gap y_t^*, foreign interest rates r_t^* and foreign inflation π_t^* are assumed to follow exogenous AR(1) processes.

The structural model can be summarised as:

$$A_0 Y_t = A_1 Y_{t-1} + A_2 E_t Y_{t+1} + \epsilon_t, \tag{6}$$

where $Y_t = [y_t, r_t, \pi_t, q_t, s_t, r_t^*, y_t^*, \pi_t^*, \psi_t, z_t, U_{q,t}]$ is a 11×1 vector containing the state variables of model and $\epsilon_t = [\epsilon_{z,t}, \epsilon_{r,t}, \epsilon_{\pi,t}, \epsilon_{q,t}, \epsilon_{s,t}, \epsilon_{r^*,t}, \epsilon_{y^*,t}, \epsilon_{\pi^*,t}]$ is an 8×1 vector of structural innovations.[7] The solution of the model can be represented as a first order VAR:

$$Y_t = B_1 Y_{t-1} + B_2 \epsilon_t. \tag{7}$$

(*i*) *Data Description*

Data from 1980Q1 to 2006Q1 for the Australian economy is used to estimate the structural model and the VAR.[8] The starting period coincides with previous SVAR studies of the Australian economy including Dungey and Pagan (2000). Quarterly observations on real total GDP (y_t), quarterly headline CPI inflation (excluding interest rates and taxes) (π_t), the (goods and services) terms of trade (s_t), the real exchange rate (q_t), the nominal interest rate (measured

[7] In the numerical simulation and estimation of the model, the structural equation is solved using a solution algorithm described in Uhlig (1995).

[8] The effective sample period is from 1980Q4 to 2006Q1 after differencing and construction of the cyclical component of GDP.

by the 90-day bank bill rate) (r_t), US GDP (y_t^*), US CPI inflation quarter-on-quarter (π_t^*) and US nominal interest rate (r_t^*) are sourced from the Reserve Bank of Australia, the Australian Bureau of Statistics and the IMF's International Financial Statistics database.[9]

The cyclical component of GDP for both Australia and the US – that is, the output gap measures – are constructed using the BN decomposition described earlier. Due to the unusual upswing in Australia's terms of trade between 2004 and 2006, this time series is detrended using an HP filter to ensure stationarity of the series.[10] All variables apart from inflation and interest rates enter in logs.

(*ii*) *Estimating the DSGE Model*

The parameters of the DSGE model are estimated using constrained maximum likelihood (ML). The likelihood function is computed via the state-space representation of the model's solution in Equation (7), together with the measurement equation linking the observed data and the state vector:

$$Z_t = GY_t. \tag{8}$$

where: Z_t denotes the observed data; and the matrix G specifies the relationship between the state variables and the observed data. The posterior parameter distribution is simulated using the Metropolis Hasting (MH) algorithm described in Lubik and Schorfheide (2007).

The ML estimates are generated conditional on the OLS estimate of the model's four exogenous processes that explain developments in the rest of the world: the terms of trade s_t, foreign inflation π_t^*, interest rates r_t^* and output y_t^*. There are two advantages in estimating the observed exogenous processes independently of the model. First, it reduces the number of parameters to be estimated in the simulation algorithm. Second, Fukac and Pagan (2006) argue that rigid restrictions imposed by DSGE models on the data may yield invalid estimates of the model's observable shocks (that is, shocks that are mapped into actual data, such as the foreign output gap y_t^*).

[9] Data for the G7 economies is also taken from the IMF's *IFS* and combined using the following weights: the US (0.49); Japan (0.16); Germany (0.10); the UK (0.07); France (0.07); Italy (0.07); and Canada (0.04).

[10] The HP filter imposes the requirement that the permanent component of the ToT series is an I(2) process, that is the ToT series is subject to shifts in the trend growth rate. The HP filter was used as a simplistic assumption and close examination does reveal a small albeit insignificant upward trend in the growth rate of the ToT.

TABLE 2

Maximum likelihood estimates of the Structural model

Parameters		**MLE statistics**				**Diagnostics**	
	Mean	Std	2.5%	97.5%	NSE	p-value	B-G
$n1$	0.09	0.06	0.01	0.24	0.00	0.06	1.03
$n2$	0.01	0.01	0.00	0.04	0.00	0.75	1.00
$n3$	0.21	0.10	0.05	0.43	0.01	0.54	1.01
$n4$	0.26	0.09	0.15	0.50	0.01	0.02	1.12
$n5$	-0.70	0.16	-1.11	-0.43	0.02	0.56	1.01
$g1$	0.27	0.05	0.16	0.37	0.01	0.93	1.00
$g2$	0.01	0.01	0.00	0.04	0.00	0.09	1.01
$g3$	0.00	0.00	0.00	0.01	0.00	0.24	1.00
ρ_r	0.90	0.02	0.84	0.93	0.00	0.07	1.11
ϕ_1	1.31	0.22	1.02	1.87	0.03	0.24	1.05
ϕ_2	1.56	0.38	0.78	2.30	0.05	0.16	1.09
ρ_z	0.78	0.07	0.62	0.89	0.01	0.22	1.04
ρ_u	0.98	0.01	0.95	1.00	0.00	0.94	1.00
σ_z	2.10	0.16	1.83	2.52	0.02	0.95	1.00
σ_π	1.03	0.22	0.72	1.54	0.03	0.36	1.03
σ_r	1.10	0.08	0.97	1.28	0.01	0.92	1.00
σ_q	1.78	0.13	1.55	2.06	0.02	0.13	1.05
ρ_s	OLS	0.90					
ρ_{r^*}	OLS	0.94					
ρ_{π^*}	OLS	0.62					
ρ_{y^*}	OLS	0.29					
σ_s	OLS	1.75					
σ_{r^*}	OLS	1.07					
σ_{π^*}	OLS	1.67					
σ_{y^*}	OLS	4.14					

Notes: (a) The posterior statistics are computed based on 1 million MCMC draws after a 50% burn in period.
(b) NSE refers to the numerical standard error of the Markov chain.
(c) p-value relates to the test of two means between the first and second half of the stationary Markov chain.
(d) B-G refers to the Brooks and Gelman (1998) univariate shrink factor. A shrink factor close to 1 is an indication of attaining a stationary distribution.

The ML estimate of the model's parameters from the 1.5 million Markov chain draws are summarised in Table 2 of the Appendix.[11] The set of Markov chain diagnostic tests imply that the simulated chains attain their stationary distributions.[12] The degree of backward lookingness is estimated to be 0.09 for the IS equation (n_1) and 0.27 for the Phillips curve (g_1). The estimated

[11] A 50 per cent burn-in is discarded before computing the summary statistics.

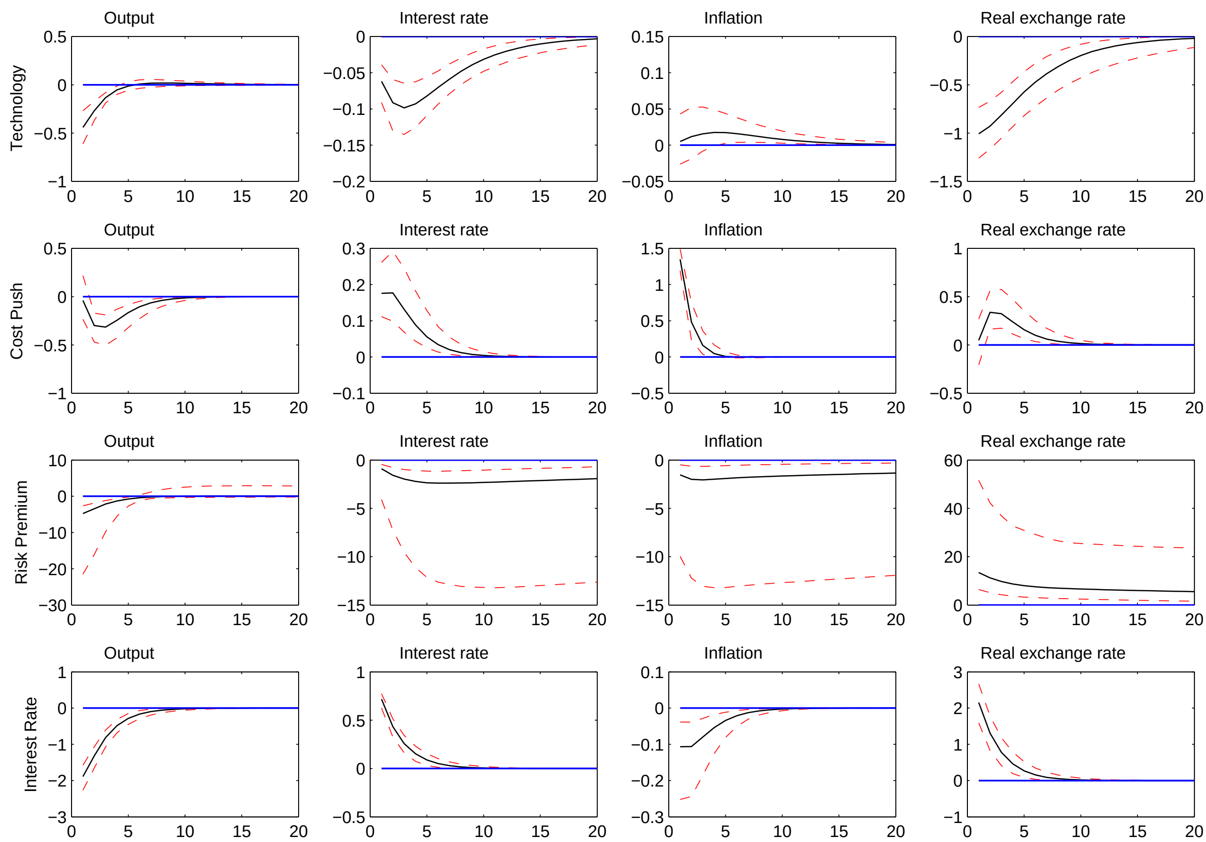
[12] There is only one exception, n_4, which is significant at the 5 per cent level. However, a small Brooks and Gelman statistic of 1.12 indicates that the chain has converged.

coefficient on the real interest rate (n_2) in the IS equation is relatively small suggesting output variation is relatively insensitive to interest rate changes. The response of inflation to output gap changes (g_2) is also estimated to be low. The Taylor rule displays a significant degree of interest rate smoothing behaviour with ρ_r estimated to be 0.90. The estimated weight on output is slightly higher than the weight on inflation and consistent with standard calibrated values used in the literature. However, the estimation covers a period before the inflation targeting regime, it is no surprise that there is a wide confidence interval around the Taylor-rule coefficient on output, ϕ_2.

(iii) Qualitative Analysis of the DSGE Model's Impulse Response Functions

FIGURE 2

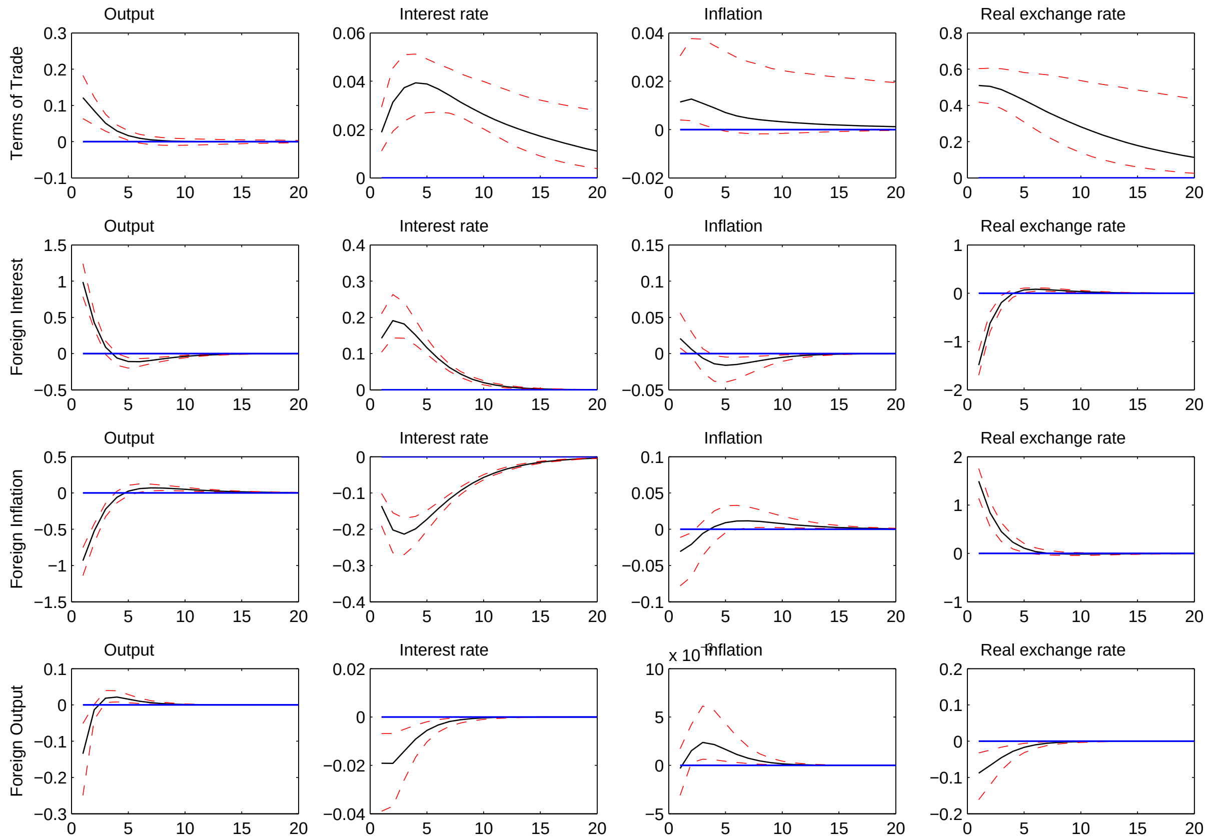
Structural model IRF of technology, monetary policy, cost push, risk premium shock.



This section present the impulse response functions of the model. The IRFs are simulated by sampling the empirical distribution of the estimates of the DSGE model. This takes into account the uncertainty of the responses associated with parameter uncertainty. The median (solid lines)

FIGURE 3

Structural model IRF of TOT, foreign interest, inflation and output shock.



along with the 5th and 95th percentile (dotted lines) responses are shown in Figures 2 and 3. The IRFs of the model are broadly consistent with other open economy studies based on New Keynesian models. Moreover, the initial responses of key variables are generally quantitatively significant providing a useful set of robust sign restrictions for the VAR analysis. The discussion here will focus more on the initial responses rather than the dynamic adjustments to the shocks.

A positive technology shock will increase the potential output of the economy (τ_t in equation 1), because of nominal rigidities actual output will take time to adjust to the new higher potential level of output, the output gap falls. This causes the interest rate to fall. The real exchange rate depreciates to reflect the change in the interest rate differential, which contributes to a small increase in the inflation rate, despite the boost to productivity.

A cost push shock increases inflation, and leads to an increase in interest rates that causes the exchange rate to appreciate and output to contract.

A negative shock to the risk premium causes lower inflation and output, due to an appreciating exchange rate. The central bank responds by reducing the interest rate. An unexpected tightening of monetary policy has a negative effect on the output gap, with lower inflation and an appreciated exchange rate.

Turning to external factors, following a positive shock to Australia's terms of trade, the output gap increases, the real exchange rate appreciates, and inflation and interest rates rise. An exogenous increase in the foreign interest rate leads to a depreciation of the domestic currency, which is sufficient to raise the output gap and together these forces push up inflation. Given the simple structure of the model, an increase in foreign inflation has a similar but opposite effect on the domestic economy as increases in the foreign nominal interest rate. An increase in foreign output actually decreases the domestic output gap, while both domestic inflation and interest rates stay relatively static and the depreciating exchange rate helps balance the international consumption risk sharing condition.[13]

(*iv*) *Robust Sign Restrictions*

The focus of the study is to gather a set of sign restrictions from the impulse responses of the DSGE model to identify the structural shocks of a small open economy VAR. The complete set of estimated IRFs from the DSGE model provides more sign restrictions than are necessary to disentangle the eight structural shocks. Bearing in mind the potential misspecification issue, not all the restrictions from the model are imposed upon the VAR. One can think of the set of sign restrictions imposed in the paper as the minimum set of restrictions needed to disentangle the eight shocks. The chosen set of sign restrictions is broadly consistent with restrictions implied by other standard open economy structural models.[14] The set of sign restrictions adopted are presented in Table 3.

There are a few important things worth highlighting. First, given that the three foreign variables enter the structural model as exogenous driving processes, the set of sign restrictions imposed on the foreign economy follows the dynamic responses implied by a canonical closed economy New Keynesian model. The responses of the domestic variables to the three foreign

[13] Gali and Monacelli (2005) provide a detailed account of the way in which such a shock can lower domestic potential output.

[14] Additional and/or alternative choices can potentially give different impulse response functions for the VAR, in particular the response to domestic shocks. However, the results of international shocks (the key focus of the paper) should remain fairly robust because there are no explicit restrictions imposed on the domestic economy from international shocks.

TABLE 3
SRVAR sign restrictions

	r^*	y^*	π^*	y	r	π	q	s
Foreign interest	↑	–	↓	–	–	–	–	–
Foreign output	↑	↑	–	–	–	–	–	–
Foreign inflation	↑	↓	↑	–	–	–	–	–
Output (composite)	0	0	0	↑	↑	–	–	–
Monetary policy	0	0	0	–	↑	↓	–	–
Cost push	0	0	0	↓	↑	↑	↑	–
Risk premium	0	0	0	–	–	↓	↑	–
Terms of trade	0	0	0	–	↑	↑	↑	↑

shocks are left unrestricted. Second, the terms of trade is treated as an endogenous variable and its response to other shocks apart from the output shock are also left unrestricted.[15] With the presence of sticky home prices in the short run, the terms of trade responds to other variables in the system via changes to domestic inflation. Third, the output shock can be viewed as anything that moves output and interest rates together but is orthogonal to all other shocks in the system. Lastly, the sign restrictions are imposed for the initial two quarters only.

IV Estimating a VAR model

This section sets out the small open economy sign restricted VAR model estimated using the data described in Section *(i)*. An eight-variable VAR(2) is fitted to quarterly observations from 1980Q4 to 2006Q1 where the number of lags are determined by the Akaike Information Criteria.

Consider a general VAR(p) model with n variables Y_t:

$$BY_t = A(L)Y_{t-1} + \epsilon_t, \qquad (9)$$

where: $A(L) = A_1L + \cdots + A_pL^p$ is an p^{th} order matrix polynomial; B is an $(n \times n)$ matrix of coefficients that reflect the contemporaneous relationships among Y_t; and ϵ_t is a set of $(n \times T)$ normally distributed structural disturbances with mean zero; and variance covariance matrix Σ, $\Sigma_{i,j} = 0 \ \forall i \neq j$. The structural representation in Equation (9) has the following reduced form:

$$Y_t = \Pi(L)Y_{t-1} + e_t, \qquad (10)$$

[15] The terms of trade is defined here as the domestic currency relative price of exports over imports.

where $\Pi(L) = B^{-1}A(L)$ and e_t is a set of $(n \times T)$ normally distributed reduced-form errors with mean zero and variance covariance matrix V, $V_{i,j} \neq 0 \; \forall i,j$. The aim is to map the statistical relationships summarised by the reduced form errors e_t back into economic relationships described by ϵ_t. Let $P = B^{-1}$. The reduced form errors are related to the structural disturbances in the following manner:

$$e_t = P\epsilon_t \quad \text{and} \quad V = E(e_t e_t') = HH', \tag{11}$$

for some matrix H such that $HH' = P\Sigma P'$. An identification problem arises if there are not enough restrictions to uniquely pin down H from the matrix V.[16]

(*i*) *Identification Through Sign Restrictions*

The identification of structural shocks is often a controversial issue, with different identifying assumptions leading to quite different conclusions. Typical restrictions employed in the literature are based on restricting the short-run or long-run impact of certain shocks on a subset of variables to be zero. The Choleski decomposition is an example of one such strategy where the contemporaneous impact of shocks follows a recursive ordering. One noticeable feature of standard empirical DSGE models is that they almost never imply zero contemporaneous impacts. This is also the case with the estimated structural model presented in Section *III*.

The central idea behind structural VAR analysis is to decompose the set of reduced form shocks, characterised by V, into a set of orthogonal structural disturbances characterised by Σ. However, there are an infinite number of ways in which this orthogonality condition can be achieved. Let H be an orthogonal decomposition of $V = HH'$. The multiplicity arises from the fact that for any orthonormal matrix Q (where $QQ' = I$), such that $V = HQQ'H' = \tilde{H}\tilde{H}'$ is also an admissible decomposition of V. This decomposition does not have any economic content, but nevertheless, produces a set of uncorrelated shocks $\epsilon_t = \tilde{H}e_t$, without imposing any zero contemporaneous restrictions.

The identification strategy used here closely follows Canova and De Nicolo (2002), Uhlig (2005), and Peersman (2005) in using qualitative information directly from IRFs to achieve identification. Canova and De Nicolo (2002) proposed an algorithm to trace out all possible

[16] There are n^2 unknowns elements in H with only $n(n+1)/2$ unique elements in V.

orthogonal vector moving average (VMA) representations of the VAR consistent with a given set of sign restrictions. See the Appendix for a more detailed description of the algorithm.

(ii) Finding the Median Impulse

The next step is to construct a summary measure from all the VAR representations consistent with the given set of sign restrictions. A common approach is to examine all of the feasible IRFs implied, and report the median response at each horizon for each variable. However, Fry and Pagan (2007) criticise this approach since the implied 'median' IRF may not actually be a feasible response (since it is likely to consist of selected parts of paths implied by different candidate functions). In other words, inference is difficult because the orthogonality condition may be violated.

Fry and Pagan suggest locating a unique identification matrix such that all of the feasible impulses are closest to its median while maintaining the orthogonality condition. Each feasible VAR representation can be distinguished by the rotation angle, θ. So the objective is to choose θ so as to minimise:

$$\Upsilon(\theta_j) = \sum_{i=1}^{q}(\phi_i^j - \bar{\phi}_i)(\phi_i^j - \bar{\phi}_i)', \tag{12}$$

where: the index i refers to the horizon for which the impulses are calculated; ϕ_i^j is an $n \times n$ matrix of standardised impulses for the jth rotation; and $\bar{\phi}_i$ is the median impulse over all possible rotations.[17] Full details of the methodology and implementation are provided in the Appendix.

V Sign-restricted VAR Results

The identification scheme based on the sign-restricted VAR allows for a structural interpretation of the effects of shocks. The impulse response of the output gap, the interest rate, inflation, the real exchange rate and the terms of trade with respect to the three foreign shocks are shown in Figure 4. An exogenous increase in the foreign interest rate results in a depreciation of the exchange rate which raises domestic inflation. In contrast to the DSGE model, the depreciation of the exchange rate is more gradual, reaching a peak at 8 quarters before returning to equilibrium. A more important difference from the DSGE results is that output falls, which

[17] In Fry and Pagan (2005), q is set to 1 focusing only on the initial period impulse.

appears to reflect the decline in foreign output (not shown) and would also help to explain why domestic interest rates decline.

In contrast to the DSGE estimates, the sign-restricted VAR estimates imply that an increase in foreign output leads to a positive domestic output gap, reaching a peak after 4 quarters. The positive domestic output gap implies increased inflationary pressure, which induces a tightening of monetary policy over time to bring both output and inflation back to steady state. The response of the domestic economy following a foreign inflation shock is very similar to that implied by the DSGE model. The exchange rate appreciates in response to the lower real interest rate differential. This leads to a fall in the output gap and subsequently a decline in inflation. There is a small monetary loosening to bring both output and inflation back to equilibrium.

Figures 4 and 5 display the summary IRFs from the sign-restricted VAR for the remaining five domestic shocks. A positive output shock (that is, a negative technology shock) raises the interest rate consistent with the sign restriction. This shock also induces inflationary pressure and the interest rate remains above its steady state level for some time. An unanticipated tightening of monetary policy lowers both inflation and output while the exchange rate appreciates in response to higher real interest rates. After the shock, the interest rate falls so as to stimulate output and bring inflation back to its steady-state level. Following a positive cost-push shock, the domestic interest rate increases, the exchange rate appreciates and the output gap falls. A negative shock to the risk premium triggers an appreciation of the exchange rate leading to lower inflation. The monetary authority responds to this by lowering the domestic interest rate. In contrast to the structural model, the effect of the monetary response is estimated to outweigh the effect of the higher exchange rate, leading to higher output. A terms of trade shock has a positive effect on both output and inflation leading to a tightening of monetary policy. The exchange rate also responds to the higher terms of trade, helping to stabilise both output and inflation.

The results highlight important differences between the responses of the SRVAR compared with the estimated DSGE model. This further emphasizes Del Negro and Schorfheide's 2009 conclusion that model misspecification remains the key challenge in applied macroeconomic research. However, this is not to say that all the restrictions are invalid; some remain useful especially when the data can not be used to help distinguish competing theories as in the case of exactly identified VARs. Peersman and Straub (2009 forthcoming) use a similar approach to try and disentangle the response of hours worked following a productivity shock in the euro area.

(*i*) *Main Drivers of Output Over the Business Cycle*

Variance decompositions are often used to determine the relative contribution of shocks to the forecast error variance of a variable of interest over different horizons. As a benchmark, I first present a variance decomposition based on the Choleski decomposition. The variables are ordered according to the convention that the most exogenous (or predetermined) variables appear first. The variance decomposition results reported in Table 4 are based on the following ordering: foreign output, foreign inflation, the foreign interest rate, the terms of trade, the output gap, inflation, the interest rate and the real exchange rate. Investigation of other ordering schemes, where the order of output among the domestic variables varies from first to last, reveals little difference in the variance decomposition results for output. The benchmark results show that at the one-year horizon, shocks to the domestic output gap account for around two thirds of the total variance in the output gap while other domestic factors play only a modest role. Foreign shocks account for just over one quarter of the output gap forecast error variance, with the biggest contributor being foreign output accounting for around 16 per cent. At longer horizons, the role of domestic output shocks decreases slightly while other domestic factors play a slightly larger role. The contribution from all foreign factors stays fairly constant across the different forecasting horizons.

Looking at the variance decomposition of the shocks identified by the sign-restricted VAR model reveals some important differences (Table 5). These results suggest that domestic output shocks only account for 4-5 per cent of the variation across all horizons. At the shorter horizons, all three foreign factors combine to account for more than 60 per cent of the output gap forecast error variance. A sizeable share of this appears to be due to foreign monetary policy innovations, although this may, in part, reflect factors that are outside of the model, such as global confidence, that are transmitted to the domestic economy via international financial markets. This view is consistent with the findings in Dungey and Pagan (2000), which shows that international financial linkages are important when modelling the Australian economy. At the longer forecasting horizon, all three foreign factors maintain their influence on domestic output gap variations with both the foreign interest rate and foreign output remaining the dominant contributors.

TABLE 4

Baseline Choleski variance decomposition of output, interest, inflation and real exchange rate

Horizon	Foreign interest	Foreign output	Foreign inflation	Output	Interest	Cost push	exchange rate	TOT
				Output				
1	0.3	2.0	7.5	89.0	0.0	0.0	0.0	1.2
4	3.3	15.7	7.5	65.8	0.9	3.5	2.3	1.0
8	3.1	14.8	8.9	62.1	1.9	4.5	3.3	1.4
12	3.0	14.5	9.1	60.1	2.8	5.3	3.7	1.5
50	3.0	14.4	9.1	58.6	3.8	5.8	3.8	1.6
				Interest rate				
1	0.7	0.1	1.2	3.7	90.5	1.6	0.0	2.1
4	1.9	9.8	5.3	17.2	53.7	10.8	0.7	0.7
8	5.1	16.7	4.5	24.9	35.7	12.3	0.3	0.4
12	6.4	19.7	3.7	28.4	29.2	11.5	0.4	0.7
50	8.9	23.0	2.9	29.1	22.5	9.1	1.1	3.4
				Inflation				
1	3.0	0.9	0.1	0.7	0.0	94.9	0.0	0.4
4	2.7	2.4	1.8	0.9	12.1	74.1	4.0	2.0
8	3.7	5.8	2.2	4.4	12.5	64.3	4.2	3.0
12	5.1	8.4	2.0	6.8	11.6	58.2	4.5	3.4
50	7.4	11.7	1.8	8.8	10.3	50.8	4.6	4.6
				Exchange rate				
1	0.7	0.1	0.0	0.1	3.0	0.0	87.8	8.2
4	0.7	0.1	6.8	3.2	2.8	3.9	80.5	2.1
8	4.5	1.2	14.3	2.2	2.8	5.3	68.0	1.7
12	9.3	2.8	17.6	1.7	6.5	5.3	55.1	1.7
50	11.7	4.0	17.0	1.8	13.5	5.4	44.6	2.0

Although the model treats the terms of trade as endogenous, realistically it can be thought of as exogenous, at least over longer horizons. So in this respect the terms of trade could be thought of as another foreign factor. The terms of trade account for a quarter of the variation in output across all but the shortest of horizons. This is consistent with the significance of commodities in Australian exports. Turning to domestic factors, interest rate shocks are estimated to have only a small influence on output gap fluctuations, while inflation (cost-push) and exchange rate (risk premium) shocks each contribute around 5-8 per cent to the variance of the output gap. This is broadly similar to the Choleski baseline results.

One may ask what is the role of foreign factors among other admissible rotations since it is impossible to distinguish them statistically. To check the sensitivity of the variance decomposition results around the optimised median impulse, the chosen median rotation is dropped and the next median impulse is found by re-optimising Equation (20) over the remaining admissible

TABLE 5

SRVAR variance decomposition of output, interest, inflation and real exchange rate

Horizon	Foreign interest	Foreign output	Foreign inflation	Output	Interest	Cost push	exchange rate	TOT
				Output				
1	49.1	0.0	1.5	5.2	0.2	0.7	7.4	35.9
4	41.7	17.1	1.8	4.4	0.5	5.2	4.8	24.5
8	40.5	17.0	1.9	4.4	0.5	6.6	4.8	24.3
12	40.1	16.7	1.9	4.3	0.5	7.2	4.8	24.6
50	39.4	16.6	2.0	4.4	0.5	7.5	5.0	24.6
				Interest rate				
1	2.4	2.2	5.6	11.1	3.1	6.6	59.4	9.5
4	1.8	3.7	3.6	8.1	1.4	13.7	46.4	21.2
8	1.6	20.3	1.9	7.3	1.4	10.3	34.4	22.6
12	1.3	32.5	1.5	7.2	1.4	8.0	28.9	19.2
50	1.1	54.7	1.6	5.2	1.0	5.5	18.6	12.3
				Inflation				
1	25.7	0.8	1.2	0.1	8.2	63.1	0.5	0.4
4	19.6	3.1	1.6	3.7	6.0	44.2	19.8	2.1
8	16.2	11.9	1.7	6.1	4.9	37.2	19.2	2.8
12	14.1	21.4	1.5	6.5	4.3	32.3	17.5	2.5
50	10.8	38.9	1.8	5.1	3.3	24.6	13.2	2.3
				Exchange rate				
1	0.5	2.2	23.3	43.9	2.7	1.0	26.2	0.1
4	8.6	3.2	18.9	27.6	3.2	6.1	30.2	2.3
8	13.2	11.9	12.7	20.2	4.0	12.6	21.1	4.4
12	13.5	25.8	8.8	13.8	3.5	14.0	14.3	6.3
50	10.7	40.0	7.8	9.4	2.5	11.9	10.1	7.5

rotations. Repeating this procedure 50 times around the 'median region' reveal foreign factors explains between 45 per cent to 60 per cent of the unconditional variance in output, with foreign interest rates remaining the dominant contributor. To give a more complete picture, Figure 6 plots the forecast error variance for the output gap attributed to foreign factors at both the one-year and 50-quarter horizon across all 2000 admitted rotations.[18] The first point to note is that the results presented above lie exactly on the mode of the distribution, while the baseline Choleski decomposition lies in the thin tail of the distribution. Looking at the range of values from the sign-restricted VAR analysis, it appears that the true importance of foreign factors may not be easily captured by Choleski decompositions that impose contemporaneous (zero) coefficient constraints.[19]

[18] The contribution from domestic factors can be easily read off the graphs since the two factors must sum to 100.

[19] Estimating the sign restricted VAR over the shorter sample 1992:Q1 - 2006:Q1 suggests that if anything foreign output shocks have become more important for explaining the variance of the domestic output gap, while

Variance decompositions may reveal which shocks are important at explaining the forecast errors of output across different horizons. However, Fry and Pagan (2005) argue they may not be very useful in understanding the nature of business cycle fluctuations. One useful statistic is to decompose the historical observation of output into its MA representation in terms of shocks, that is:

$$y_t = \sum_{j=1}^{k} C_j(L)\epsilon_{j,t} + \text{initial condition}, \tag{13}$$

where $C_j(L)$ is the impulse response to the shock j.[20] Historical decompositions are particularly useful in relating certain events that have happened over the business cycle.

Figure 7 plots the historical decomposition of output into foreign (output, inflation and interest rates) versus domestic factors. During the two recessionary periods (the early 1980s and 1990s), both domestic and foreign factors had contributed negatively to output. This observation is consistent with the results reported in Dungey (2002). From the early 1990s onwards, the Australian economy experienced relatively stable and low inflation combined with robust output growth. Coincidentally, foreign and domestic shocks appear to have had offsetting effects so as to moderate domestic business cycle fluctuations during this period. For example, the slowdown in the economy after the Sydney Olympic games together with the introduction of GST in 2000 was somewhat offset by buoyant conditions before the bursting of the 'dot-com' bubble in the US. A buoyant housing market and strong household consumption in the early part of this decade was moderated by a temporary downturn in the US economy following the terrorist attacks in September 2001. The pattern continued in late 2003 where slowing conditions in the Australian housing market were offset somewhat by a relatively strong US economy.

(*ii*) *Robustness checks*

G7 as the Foreign Economy

To check the robustness of these results, the sign-restricted VAR model is re-estimated using G7 data as the foreign economy. It is certainly the case that China has become increasingly more important for the Australian economy, as of 1 October 2009 the RBA calculates the share

shocks to foreign interest rates have become less so. However, this sample may be too short to produce reliable estimates of the relatively high dimensional VAR.

[20] Since the entire history of shocks are not observed, the decomposed components of y_t may not add up exactly for the initial periods of the sample. In the case of output, this is around 6-8 quarters, which are dropped from the decomposition results shown in Figure 6 below.

of the Chinese currency in the Trade weighted index (TWI) to be over 16%. However, over a large part of the sample this paper considers, the G7 countries remained as Australia's dominant trading partners. According to the RBA Bulletin (2002), the weight of the Chinese currency in Australia's TWI ranged from 2% in 1980 to 8.5% in 2002, whereas the G7 countries accounted for 77% to 52% over the same period.

The overall conclusion using the G7 dataset is supported although minor differences do arise.[21] The combined contribution of foreign shocks accounts for around 63 per cent of the forecast error variance for the output gap at the one-year horizon, similar to that reported earlier. At the 50-quarter horizon, this increases to 76 per cent in contrast to 59 per cent based on using only US data. Consistent with the earlier estimates, innovations from domestic output play a smaller role in explaining domestic output gap forecast errors. However, within the set of international variables, foreign output now takes on a larger role compared with foreign interest rates. This tends to suggest that interest rates may have been picking up other global factors in the results based on US output alone.

Using the HP filter as alternative detrending method

Another important assumption behind the discussions above is that the results are based on cyclical movements extracted from the BN decomposition. As an additional robustness check, the VAR is re-estimated using cyclical movements extracted from the HP filter.

Across all horizons, the three foreign shocks together contribute around 50% of the forecast errors for output, whereas domestic output shocks only account for under 9%. In contrast to the BN decomposition, foreign output shocks are now the dominant contributor at around 30% and foreign inxterest rate shocks around 17%. The results demonstrate that the key determinant of the output forecast error rests with the sign restriction identification.

VI Conclusion

This paper uses a small open economy VAR model to investigate the sources of business cycle fluctuations for the Australia economy. The VAR is identified using robust sign restrictions derived from an estimated small structural (DSGE) model. The results suggest that international

[21] Detail statistics are not reported but are available upon request.

factors account for over half the domestic output fluctuations while demand type shocks play a small role. The result appears to be robust to alternative detrending of the data and using different representation as the foreign economy.

The paper tries to address some of the shortcomings in the earlier sign restriction literature that the sign restrictions are arbitrarily imposed by selecting a set of robust sign restrictions from an estimated structural model. However, the analysis does not tackle the question which model is more valid and this issue is left to future work.

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Appendix: Sign Restriction Algorithm

Define an $(n \times n)$ orthonormal rotation matrix Q such that:

$$Q = \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} Q_{i,j}(\theta_{i,j}), \tag{14}$$

$$\text{where} \quad Q_{i,j}(\theta_{i,j}) = \begin{bmatrix} & & & \text{col } i & & \text{col } j & & \\ & 1 & & \downarrow & & \downarrow & & \\ & & \cdots & & & & & \\ \text{row } i & \rightarrow & & \cos(\theta_{i,j}) & \cdots & -\sin(\theta_{i,j}) & & \\ & & & \cdots & 1 & \cdots & & \\ \text{row } j & \rightarrow & & \sin(\theta_{i,j}) & \cdots & \cos(\theta_{i,j}) & & \\ & & & & & & \cdots & \\ & & & & & & & 1 \end{bmatrix},$$

where $\theta_{i,j} \in [\ 0, \pi\ ]$. This provides a way of systematically exploring the space of all VMA representations by searching over the range of values of $\theta_{i,j}$. While Canova and De Nicolo (2002) propose setting a up grid over the range of values for $\theta_{i,j}$, the following algorithm generates the Q's randomly from a uniform distribution:

1. Estimate the VAR in Equation (10) using OLS to obtain the reduced form variance co-variance matrix V and compute $\tilde{V}$;

2. Compute the Choleski decomposition of $\tilde{V_{11}}$ and $\tilde{V_{22}}$, where $H_{11} = chol(\tilde{V_{11}})$ and $H_{22} = chol(\tilde{V_{22}})$;

3. For both the foreign and domestic block, draw a vector of $\theta_{i,j}$ from a Uniform $[\ 0, \pi\ ]$ distribution;

4. Calculate $Q = \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} Q_{i,j}(\theta_{i,j})$;

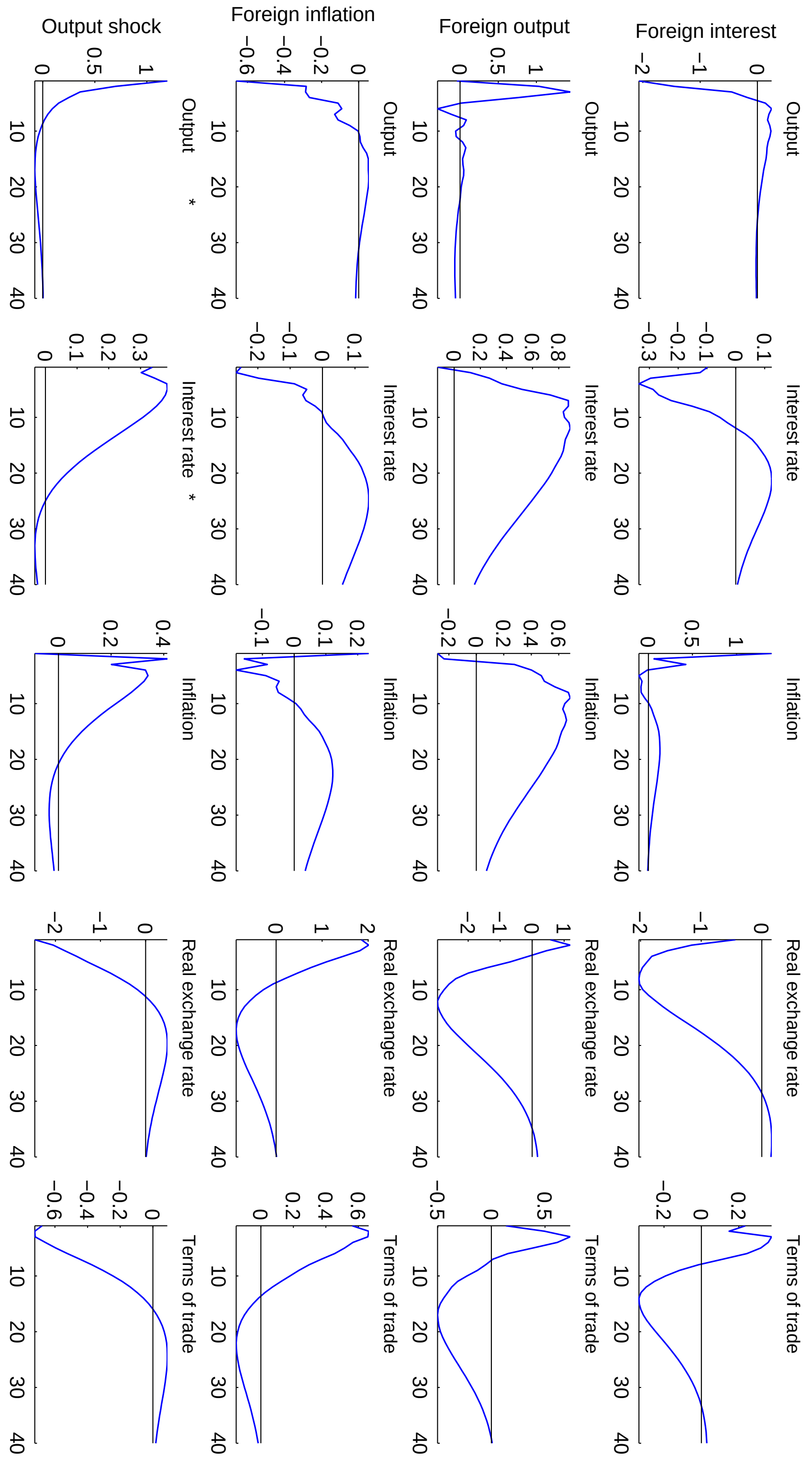
5. Use the candidate rotation matrix Q to compute $\epsilon_t = HQe_t$ and its corresponding structural IRFs $C(L)$ for domestic and foreign shocks;

6. Check whether the IRFs satisfy all the sign restrictions described in Table 3. If so keep the draw, if not, drop the draw;

7. Repeat (3)–(6) until 2000 draws satisfying the restrictions are found.

FIGURE 4

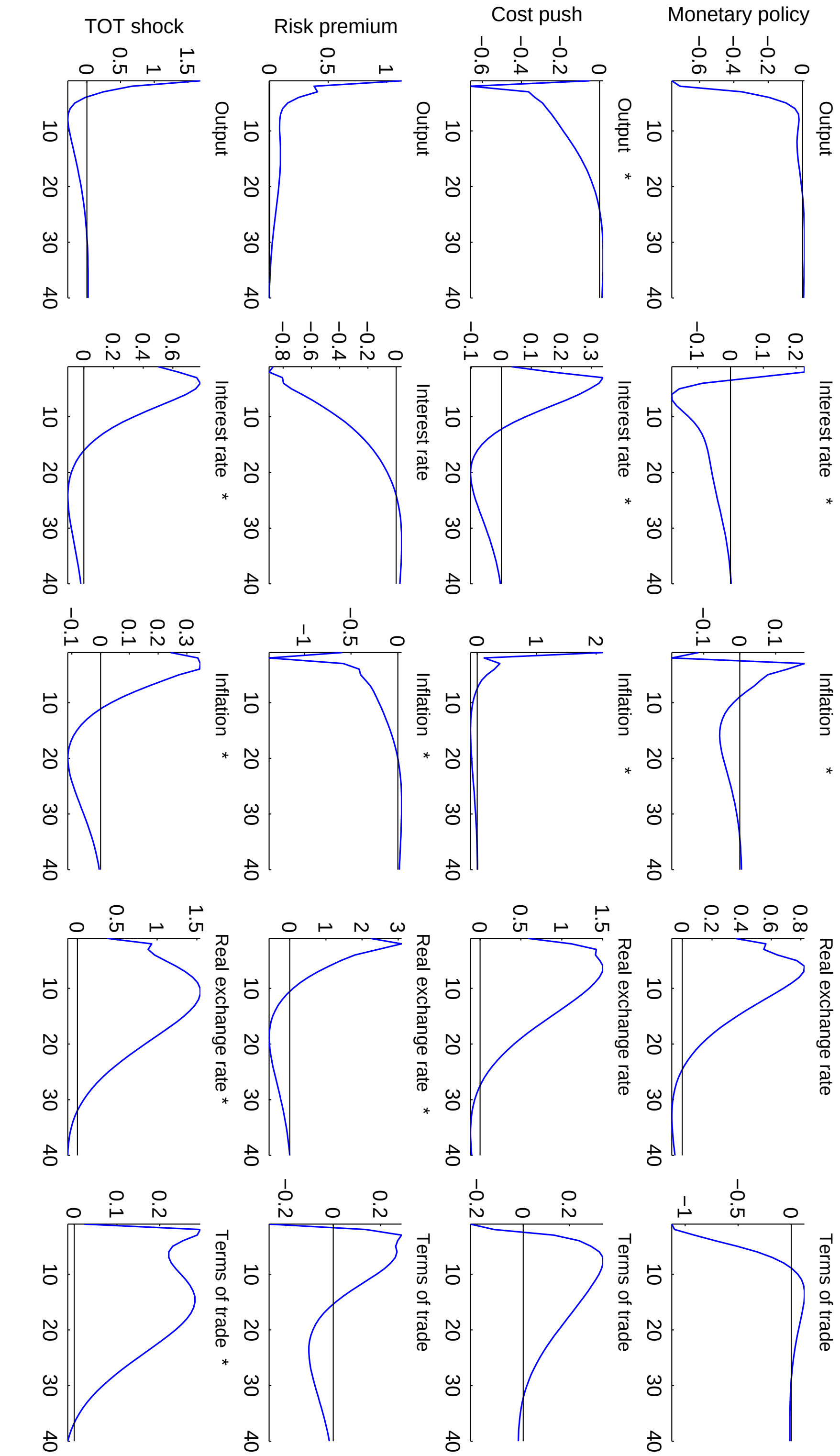
SRVAR IRF of foreign output, interest, inflation and domestic output shock.



Note: * indicate impulse responses where sign restrictions are imposed.

FIGURE 5

SRVAR IRF of monetary policy, cost push, risk premium and TOT shock.



Note: * indicate impulse responses where sign restrictions are imposed.

FIGURE 6

SRVAR Variance Decomposition of Foreign vs Domestic Factors Across 2000 Rotations.

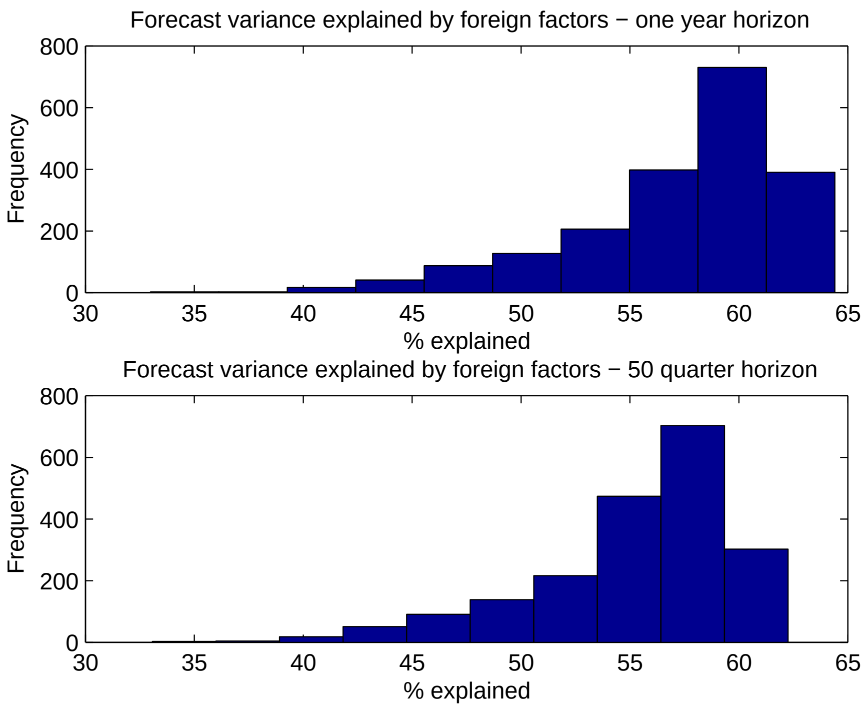


FIGURE 7

Historical Decomposition of Output Using US data.